On the Mathematical Model of the Biomechanics of Green Plants

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Authors' contributions

This work was carried out in collaboration between both authors. Both authors read and approved the final manuscript.

ABSTRACT

This study considers the biomechanics in the stem of green plants. The process of translocation and transpiration is discussed. The coupled non-linear differential equations governing the motion of the flow were non-dimensionlized and then solved using the homotopy perturbation method. The effects of various parameters such as Schmidt number, porosity, buoyancy forces (thermal and concentration Grashof numbers) and aspect ratio embedded in the flow were examined on the concentration field. The results showed that increasing the porosity, Schmidt number, Sherwood number and aspect ratio resulted to a decrease in the concentration field whereas increase in the buoyancy forces had a positive effect on the flow by increasing its concentration and hence enhancing the growth and productivity of the plant.

Keywords: Biomechanics; xylem flow; phloem flow; Homotopy Perturbation Method (HPM).

1. INTRODUCTION

Biomechanics consist of the word “bio” meaning life or living thing and “mechanics” a branch of physics that describes motion and how forces create motion. Thus, [1] defined biomechanics as “the study of the movement of living things using the science of mechanics”. Hence we shall consider the biomechanics of green plants.
Green plants are among the most successful organisms on earth in terms of biomass and individual size range [2]. They are the earth's primary solar energy collectors and are the ultimate source of food for both man and animals [3].

The mechanics of green plants when compared to that of animals is open. In the sense that it consists of the roots, stems and leaves. The roost absorbs water and minerals from the soil and transports it through the stem and then to the leaves. The stem consists basically two vascular tissues, namely xylem and phloem [4, 5]. The xylem is made up of the tracheids and vessel elements that die after reaching maturity while the phloem contains sieve elements that still lives after maturity [3]. The water and minerals that enters the stem are transported upwards through the xylem and then to the leaves through its petiole. Most of the water leaving the xylem (tracheary elements) moves into the leaf mesophyll (sieve elements) and then evaporates into the atmosphere through the stomata (this process is called transpiration). Carbon dioxide enters the leaves from the atmosphere through the stomata by diffusion and then combines with part of the water that entered the mesophyll cells in the presence of sunlight to form carbohydrate by the process of photosynthesis. The carbohydrate produced is pushed downward with the aid of water into the phloem vessel and then translocated downwards to the fruits, shoots and roots where they are needed (this process is known as translocation) [6,2,3,7].

The force that drives the upward flow in the xylem is enhanced by suction pressure generated in the leaves by evaporation of water vapor into the atmosphere [2] and the environmental thermal differences resulting from free convective motion of the fluid [8]. The downward phloem on the other hand is driven by concentration differences resulting from active transport [3].

From literature, it is observed that fluid carrying vessels of green plants are porous and the flow naturally convective. Studies have also shown that flow through porous channels are affected by certain parameters such as, thermal gradient, concentration gradient, pressure gradient (for example suction and root pressure), the porosity, permeability, the physical properties of the fluid (for example, viscosity, density), body forces (for example, gravity, magnetic field, buoyancy force) [9,10,11,12] etc. Several methods such as Laplace transform, perturbation, direct numerical simulations have also been used to examined the effects of these parameters on the flow structure.

Bestman [6] examined the effect of small value of the aspect ratio and increasing values of the porosity on the concentration field of a fully developed flow using Laplace transform method. Bestman [13] went further to consider the case where the flow is not fully developed for larger value of the aspect ratio using perturbation and finite Fourier sine and cosine technique. The effects of heat source and magnetic field on unsteady MHD blood flow through bifurcated arteries was examined by Prakash et al. [14]. On the other hand, Okuyade [15] studied MHD blood flow through bifurcated porous fine capillaries of humans using perturbation method. Effects of magnetic field and environmental thermal parameters on the flow structure were examined. Tadjar and Smith [16] examined the effects of bifurcation angle on a 3-dimensional laminar steady flow of an incompressible viscous fluid through a straight mother tube bifurcating into two straight but divergent daughter tubes by direct numerical simulations. Liou et al. [17] studied the effect of bifurcation angles on the steady flow structure in a straight terminal aneurysm model with asymmetric outflow through the branches using the Laser-Doppler velocity and fluctuating intensity distribution. Okuyade and Abbey [18] studied a steady MHD fluid flow in a bifurcating rectangular porous medium using perturbation method. The effects of bifurcation angle, magnetic field, thermal and concentration Grashof numbers on the flow were examined. Rand and Cooke [19] studied flow through sieve tubes with sieve plates in the phloem of plants using an idealized single-pore axisymmetrical model. Rand et al. [20] also studied the flow using an approximate formula. Jensen et al. [21] presented an experimental and theoretical study of transient osmotically driven flows through pipes with semi-permeable walls. Cabrita et al. [22] studied the transport phloem which allows leakage of solute of a steady state model. The sieve tube membrane permeability strongly influenced the results of the model. Payvandi et al. [23] studied the transport of water and nutrient in xylem vessels of a wheat plant. Solution to the transport of the nutrient was obtained considering convection and diffusion.

In this study however, we will consider the effects of increasing values of the porosity, aspect ratio, Schmidt number, buoyancy forces (thermal and concentration Grashof number) on the
2. MATERIALS AND METHODS

Thus, our normalized governing equations are

\[ \rho c_p \left( u' \frac{\partial u'}{\partial r} + w' \frac{\partial u'}{\partial z} \right) = \alpha \left( \frac{\partial^2 u'}{\partial r^2} + \frac{1}{r} \frac{\partial u'}{\partial r} + \frac{\partial^2 u'}{\partial z^2} \right), \quad (4) \]

\[ u' \frac{\partial c'}{\partial r} + w' \frac{\partial c'}{\partial z} = D \left( \frac{\partial^2 c'}{\partial r^2} + \frac{1}{r} \frac{\partial c'}{\partial r} + \frac{\partial^2 c'}{\partial z^2} \right). \quad (5) \]

where \( p \) is the pressure, \( v \) is the viscosity, \( T \) and \( C' \) are the fluid temperature and concentration respectively, \( T_w \) and \( C_w \) are the temperature and concentration at equilibrium, \( K \) is the permeability, \( g \) is the gravitation which acts in opposite direction to the flow, \( \rho \) is the fluid density, \( \beta_t \) and \( \beta_c \) are the coefficient of volume expansion for temperature and concentration respectively, \( C_p \) is the heat capacity, \( \alpha \) is the thermal conductivity, \( k_0 \) is the thermal diffusivity and \( D \) is the mass diffusion coefficient.

\[ w'(0, z) = 1; \quad T(0, z) = T_w \quad C'(0, z) = C_w \quad \text{at} \quad r' = 0, \]

\[ w'(1, z) = 0; \quad T(1, z) = T_w \quad C'(1, z) = C_w \quad \text{at} \quad r' = 1. \quad (6) \]

where \( T_w \) and \( C_w \) are the constant wall temperature and concentration respectively.

The following non-dimensional quantities are used to normalize our governing equations.

\[ x^2 = \frac{r_0}{\sqrt{R}}; \quad Gr = \frac{\beta_T (T_w - T_\infty) r_0}{\rho v^2}; \quad Gc = \frac{\beta_c (C_w - C_\infty) r_0}{\rho v^2}; \quad Sc = \frac{v}{k_0}, \quad Pr = \frac{v}{k_0}, \quad Re = \frac{v r_0}{\nu}. \quad (7) \]

Thus, our normalized governing equations are

\[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial u'}{\partial r} \right) + \frac{\partial w'}{\partial z} = 0, \quad (8) \]

\[ 0 = -R \frac{\partial p}{\partial z} + \frac{\partial^2 u'}{\partial r^2} + \frac{1}{r} \frac{\partial u'}{\partial r} - u' - x^2 u' - R^2 \frac{\partial^2 u'}{\partial z^2}, \quad (9) \]

\[ 0 = -R \frac{\partial p}{\partial z} + \frac{\partial^2 w'}{\partial r^2} + \frac{1}{r} \frac{\partial w'}{\partial r} - x^2 w' + R^2 \frac{\partial^2 w'}{\partial z^2}, \quad (10) \]

\[ Pr \left( u' \frac{\partial \theta}{\partial r} + Rw \frac{\partial \theta}{\partial z} \right) = \frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r} \frac{\partial \theta}{\partial r} + R^2 \frac{\partial^2 \theta}{\partial z^2}, \quad (11) \]

\[ Sc \left( u' \frac{\partial \psi}{\partial r} + Rw \frac{\partial \psi}{\partial z} \right) = \frac{\partial^2 \psi}{\partial r^2} + \frac{1}{r} \frac{\partial \psi}{\partial r} + R^2 \frac{\partial^2 \psi}{\partial z^2}. \quad (12) \]

For convenience, we assume a solution of the form

\[ \theta^{(0)} = \theta(r) - \gamma z, \quad \psi = \psi^{(0)}(r) - \gamma z, \quad p = Kz - \frac{L}{r} x^2, \quad (13) \]
as given by Bestman [6]. Substituting the assumed solution (13) into equations (8) - (12), we have

\[
K = w^* + \frac{1}{r} w^* - x^2 w + Gr \theta^{(0)} + Gc \phi^{(0)},
\]

\[
-P_r R_y w = \theta^{(0)^*} + \frac{1}{r} \theta^{(0)^*},
\]

\[
-S_c R_y w = \phi^{(0)^*} + \frac{1}{r} \phi^{(0)^*};
\]

where \( y \) is a constant, \( R \) is the aspect ratio, \( x \) is the porosity parameter, \( Gr \) is the thermal Grashof number, \( Gc \) is the concentration Grashof number, \( Sc \) is the Schmidt number, \( Pr \) is the Prandtl number. The transformed boundary conditions are

\[
\phi^{(0)}(0) = 1; \quad \phi^{(0)}(1) = 0,
\]

\[
w(0) = 1; \quad w(1) = 1,
\]

\[
\theta^{(0)}(0) = 1; \quad \theta^{(0)}(1) = 1.
\]

According to the HPM of [24], the homotopy form of (14), (15) and (16) are constructed as follows:

\[
(1-p) \left[ w^* \right] + p \left[ w^* + \frac{1}{r} w^* - x^2 w + Gr \theta^{(0)} + Gc \phi^{(0)} - K \right] = 0,
\]

\[
(1-p) \left[ \theta^{(0)^*} \right] + p \left[ \theta^{(0)^*} + \frac{1}{r} \theta^{(0)^*} + P_r R_y w \right] = 0,
\]

\[
(1-p) \left[ \phi^{(0)^*} \right] + p \left[ \phi^{(0)^*} + \frac{1}{r} \phi^{(0)^*} + S_c R_y w \right] = 0.
\]

We assume \( w, \theta^{(0)} \) and \( \phi^{(0)} \) as

\[
w = w_0 + pw_1 + p^2 w_2 + \cdots
\]

\[
\theta^{(0)} = \theta^{(0)}_0 + p \theta^{(0)}_1 + p^2 \theta^{(0)}_2 + \cdots
\]

\[
\phi^{(0)} = \phi^{(0)}_0 + p \phi^{(0)}_1 + p^2 \phi^{(0)}_2 + \cdots
\]

Substituting equations (21) - (23) into equation (18) and simplifying, we have

\[
w_0 + pw_1 + p^2 w_2 + p \left[ \frac{1}{r} (w_0 + pw_1 + p^2 w_2) - x^2 (w_0 + pw_1 + p^2 w_2) + Gr (\theta^{(0)}_0 + p \theta^{(0)}_1 + p^2 \theta^{(0)}_2) + Gc (\phi^{(0)}_0 + p \phi^{(0)}_1 + p^2 \phi^{(0)}_2) - K \right] = 0.
\]

Substituting equation (21) and (23) into equation (19) and simplifying, we have

\[
\theta^{(0)}_0 + p \theta^{(0)}_1 + p^2 \theta^{(0)}_2 + p \left[ \frac{1}{r} (\theta^{(0)}_0 + p \theta^{(0)}_1 + p^2 \theta^{(0)}_2) + P_r R_y (w_0 + pw_1 + p^2 w_2) \right] = 0.
\]

Substituting equations (21) and (23) into equation (20) and simplifying, we have

\[
\phi^{(0)}_0 + p \phi^{(0)}_1 + p^2 \phi^{(0)}_2 + p \left[ \frac{1}{r} (\phi^{(0)}_0 + p \phi^{(0)}_1 + p^2 \phi^{(0)}_2) + S_c R_y (w_0 + pw_1 + p^2 w_2) \right] = 0.
\]

Rearranging equations (24) - (26) based on the powers of \( p \)-terms together with its boundary conditions, we have

\[
p^0; \quad w_0 = 0; \quad w_0(0) = 1; \quad w_0(1) = 1,
\]

\[
\theta^{(0)}_0 = 0; \quad \theta^{(0)}_0(0) = 1; \quad \theta^{(0)}_0(1) = 1,
\]

\[
\phi^{(0)}_0 = 0; \quad \phi^{(0)}_0(0) = 1; \quad \phi^{(0)}_0(1) = 1.
\]
3. RESULTS AND DISCUSSION

The effects of various parameters such as the porosity parameter ($\chi$), aspect ratio ($R$), Schmidt number ($Sc$), buoyancy forces (Thermal and Concentration Grashof number, ($Gr/Sc$)) embedded in the fully developed flow with a very low Reynolds number are examined on the concentration field. Figs. 1–5 shows the results for varying values of $\chi = 1.0, 5.0, 10.0, 15.0, 20.0$, $Sc = 1.0, 5.0, 10.0, 15.0, 20.0$, $Gr = 1.0, 5.0, 10.0, 15.0, 20.0, R = 0.1, 0.5, 1.0, 5.0, 10.0$. and at fixed values of $Pr = 7.0$, $K = 0.5$ and $\gamma = 0.5$. Sherwood number effect is also shown.

It is observed from Fig.1 that for larger values of the porosity parameter, the flow concentration decreases. This agrees with the result obtained by Bestman [6]. From Figs. 2 and 3, larger
values of the aspect ratio and Schmidt number also reduced the flow concentration whereas increase in the buoyancy forces in Figs. 4 and 5 enhanced the flow concentration. And finally from Fig. 6, the Sherwood number decreases as the porosity parameter increases.

**Fig. 1.** Effects of $x$ on concentration at $Pr = 7.0, \gamma = 0.5, Sc = Gr = Gc = 1.0, R = 0.1, K = 0.5$

**Fig. 2.** Effects of $R$ on concentration at $Pr = 7.0, \gamma = 0.5, Sc = Gr = Gc = x = 1.0, K = 0.5$

**Fig. 3.** Effects of $Sc$ on concentration at $Pr = 7.0, \gamma = 0.5, x = Gr = Gc = x = 1.0, R = 0.1, K = 0.5$
Fig. 4. Effects of $Gc$ on concentration at $Pr = 7.0, \gamma = 0.5, Sc = 5.0, Gr = x = 1.0, R = 0.1, K = 0.5$

Fig. 5. Effects of $Gr$ on concentration at $Pr = 7.0, \gamma = 0.5, Sc = 5.0, Gc = x = 1.0, R = 0.1, K = 0.5$

Fig. 6. Effects of $x$ variation on Sherwood number at $Pr = 7.0, \gamma = 0.5, Sc = Gr = Gc = x = 1.0, R = 0.1, K = 0.5$
4. CONCLUSION

A steady, incompressible, two dimensional flow model on the biomechanics of green plants has just been analyzed. The coupled non-linear governing equations of the flow were non-dimensionalized and then solved by homotopy perturbation method. Analytical results for various parametric conditions of the fully developed flow were presented on the concentration field. Results showed that increasing the porosity, Schmidt number, Sherwood number and aspect ratio resulted to a decrease in the concentration flow field whereas increase in the buoyancy force had a positive effect on the flow concentration. That is to say, as the concentration or quantity of the fluid transported in the plant increases, the nutrients absorbed by the plant also increases, hence enhancing its growth and productivity.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

REFERENCES


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